LIQUID DISTRIBUTION IN WETTED SLIT REACTORS*

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Criterion equations were derived to describe the distribution of a liquid in wetted slit reactors for the cases of a dry wall, a totally reflecting wall, and a wall with a real behaviour. Relations for a semiinfinite plane system are also given.

In the preceding communication of this series, we investigated experimentally the amalgam distribution in a vertical cylindrical decomposer[1]. With respect to the fact that the width of modern amalgam electrolysers of high performance is comparable with their length[2], it is preferable to use a slit reactor[3] for the amalgam decomposition. The present paper deals with the mathematical theory of the amalgam distribution in slit reactors or, more generally, distribution of a liquid in wetted slit reactors.

Two-Dimensional System

The two-dimensional (or slit) system consists of a bed of height h (Fig. 1) between two vertical walls parallel to the yz plane in a distance a apart from this plane. The bed and the walls are considered infinite in the direction of the y axis. If the bed is wetted by an infinitely long source with a constant distribution of the liquid along the y axis, the liquid distribution depends only on the x and z coordinates, *i.e.*, the system can be considered as two-dimensional in the coordinates x and z.

The mathematical relations derived on the basis of planar geometry can be used also for the real slit system of finite length L, since they describe the real situation approximately in the case of an arbitrary behaviour of the walls (*e.g.*, s_1 and s_2 in Fig. 1) if the ratio L/a is sufficiently large, and exactly in the case of a behaviour of the walls as "total reflectors" regardless of the value of L/a.

The assumptions underlying the theory are: *1*) The bed dimensions are sufficiently large with respect to the bed particles; *2*) from the statistical point of view, the bed elements with different distribution properties are equally distributed in the bed; *3*) the influence of the wall on the liquid distribution, the so-called wall effect, is in every point of the wall the same.

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In view of the statistical character of the liquid distribution in the bed proved by several authors[4–6], it can be expected that this process is described by a partial differential equation[7] analogous to that for diffusion or heat conduction. This was proved by Cihla and Schmidt[8] who derived on the basis of certain assumptions an equation of the mentioned type for the distribution function of a cylindrical system. We can derive such an equation in an analogous way in the system of rectangular coordinates:

$$D\left(\frac{\partial^2 f(x, y, z)}{\partial x^2} + \frac{\partial^2 f(x, y, z)}{\partial y^2}\right) = \frac{\partial f(x, y, z)}{\partial z}. \tag{1}$$

The coefficient of the horizontal spreading of the liquid through the bed, D, characterizes only the influence of the properties of the bed on the liquid distribution; it is hence independent of the geometry of the system and of the wall behaviour (see further text). It is analogous to the diffusion or heat conduction coefficient.

In the case of a two-dimensional system, Eq. (*1*) takes the form

$$D\frac{\partial^2 f(x, z)}{\partial x^2} = \frac{\partial f(x, z)}{\partial z}. \tag{2}$$

The first boundary condition follows from the behaviour of the wall. A system with a dry wall is defined by the boundary condition

$$f(a, z) = 0. \tag{3}$$

This means that the bed height, h, is chosen so that the liquid attains during the distribution process the wall in a horizontal plane of the coordinate $z = h$. Relations

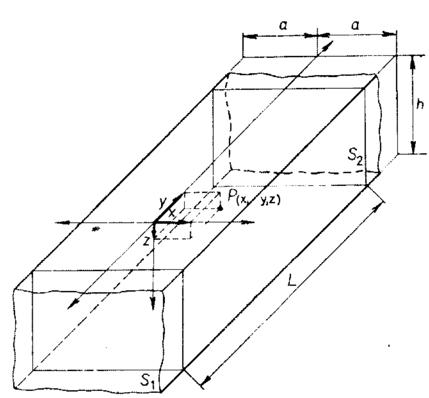

FIG. 1

Orientation of the Coordinate Axes for the Slit System

$P(x, y, z)$ origin of the x, y, z coordinates, $2a$ slit width, h slit height, L slit length.

obtained by solving Eq. (2) with the boundary condition (3) are recommended by certain authors for the determination of the distribution properties of the bed (*i.e.*, the coefficient of the horizontal spreading of the liquid through the bed, D).

A totally reflecting wall is characterized by the boundary condition

$$\left(\frac{\partial f(x, z)}{\partial x}\right)_{x=a} = 0 \,. \tag{4}$$

In comparison with the foregoing case of a dry wall, where the influence of the boundary condition with respect to the limited height of the bed is not manifested at all, the bed height h can be in the case of a totally reflecting wall arbitrary. This wall limits only the bed and does not participate in the vertical transport of the liquid; the wall flow does not take place. In the case of an even wetting of the bed limited by a totally reflecting wall, the wetting density given by its value for $z = 0$ is constant both in the vertical and horizontal directions, hence an optimum utilization of the bed surface is achieved. This theoretical model, which can be denoted as an "optimally wetted reactor", has an optimum efficiency from the point of view of the liquid distribution and corresponds to a plug flow reactor. A real wall can be approximated by a totally reflecting wall if the wall flow can be neglected (with respect to the reaction as well as to the liquid distribution). This system can be made use of in the measurement of the spreading coefficient D.

Unlike the preceding cases, a wall with a real behaviour participates in the vertical transport of liquid. On the basis of the contemporary knowledge about the liquid

TABLE I

Roots q_n of the Equation $(C/q_n - q_n/B) \sin q_n + \cos q_n = 0$

n	$C = 1$, $B = 1$	$C = 1$, $B = 3$	$C = 1$, $B = 5$	$C = 2$, $B = 1$	$C = 2$, $B = 3$	$C = 2$, $B = 5$	$C = 4$, $B = 1$	$C = 4$, $B = 3$	$C = 4$, $B = 5$
1	1·20779	1·63636	1·77583	1·46717	1·93867	2·07352	1·85784	2·32701	2·43013
2	3·44824	3·90476	4·17525	3·47485	4·01390	4·32869	3·53573	4·26278	4·64474
3	6·44095	6·72867	6·95869	6·44478	6·75581	7·01317	6·45297	6·81848	7·13936
4	9·53048	9·73295	9·91324	9·53164	9·74236	9·93526	9·53406	9·76283	9·98443
5	12·64578	12·80072	12·94526	12·64628	12·80492	12·95579	12·64729	12·81378	12·97849
6	15·77154	15·89668	16·01620	15·77179	15·89889	16·02194	15·77231	15·90347	16·03404
7	18·90256	19·00739	19·10887	18·90271	19·00869	19·11231	18·90301	19·01135	19·11946
8	22·03659	22·12673	22·21473	22·03668	22·12756	22·21694	22·03687	22·12924	22·22149
9	25·17251	25·25155	25·32913	25·17257	25·25210	25·33063	25·17270	25·25323	25·33371
10	28·30969	28·38004	28·44937	28·30973	28·38043	28·45043	28·30982	28·38123	28·45260

distribution in a cylindrical system it seems probable that the wall effect is best taken into account by the boundary condition according to Staněk and Kolář[9] resembling the theory of the convective heat transfer, or the boundary condition after Dutkai and Ruckenstein[10] based on an analogy with adsorption and desorption. These boundary conditions lead to equivalent solutions differing only by constants as a result of different qualitative concepts.

For a slit system, the boundary condition can be written in the analogous form

$$-D\left(\frac{\partial f(x, z)}{\partial x}\right)_{x=a} = \beta[f(a, z) - \gamma W(z)]\,, \tag{5}$$

which after multiplying by 2 dz represents the balance of the liquid transfer onto the wall in a system element of unit length (parallel to the y axis) and height dz in a distance z from the origin. The left-hand side of Eq. (5) gives the amount of liquid transported in a time unit from the bed to the wall, and the other side gives the same quantity based on the liquid transfer to the wall. Analogously to the theory of the convective heat transfer, the constant β is the coefficient of transfer of the liquid to the wall, and the term in brackets gives the driving force. The constant γ can be defined for $z \to \infty$ as

$$\gamma \equiv \frac{f(a, \infty)}{W(\infty)} = \frac{f(x, \infty)}{W(\infty)}\,. \tag{6}$$

The wall flow $W(z)$ for a system of unit length is given by

$$W(z) = W(0) - 2D \int_0^z \left(\frac{\partial f(x, z)}{\partial x}\right)_{x=a} \mathrm{d}z\,. \tag{7}$$

The wetting density profile for $z = 0$ is characterized by a boundary condition. For the case of a dry wall and a totally reflecting wall we shall solve Eq. (2) with the following boundary conditions: generally wetted bed:

$$f(x, 0) = \begin{cases} g(x) & x_1 < |x| < x_2 \\ 0 & 0 \leqq |x| < x_1\,;\quad x_2 < |x| \leqq a\,, \end{cases} \tag{8}$$

where $g(x)$ is any function fulfilling the Dirichlet conditions (see below);
two symmetrical band sources:

$$f(x, 0) = \begin{cases} f_0 & x_1 < |x| < x_2 \\ 0 & 0 \leqq |x| < x_1\,;\quad x_2 < |x| \leqq a\,, \end{cases} \tag{9}$$

two symmetrical linear sources:

$$f(x, 0) = \begin{cases} \infty & |x| = x_1 \\ 0 & 0 \leqq |x| < x_1\,;\quad x_1 < |x| \leqq a\,, \end{cases} \tag{10}$$

central band source:

$$f(x, 0) = \begin{cases} f_0 & 0 \leqq |x| < x_1 \\ 0 & x_1 < |x| \leqq a \,, \end{cases} \tag{11}$$

central linear source:

$$f(x, 0) = \begin{cases} \infty & x = 0 \\ 0 & 0 < |x| \leqq a \,. \end{cases} \tag{12}$$

We shall derive a solution for the system with a real wall behaviour assuming an evenly wetted bed with a given initial value of the wall flow

$$f(x, 0) = f_0 \,, \quad W(0) = W_0 \,. \tag{13a, b}$$

In the limiting cases, $W_0 = 0$ or $f_0 = 0$, we obtain solutions for the case of an evenly wetted bed or for the case of a wall source.

It follows from the boundary conditions (*8*)—(*13*) that the studied system will be in all cases symmetrical with respect to the yz plane. With regard to the assumption of continuity of the distribution function for $z > 0$ (ref.[8]), we can write the second boundary condition (denoted in further text as symmetry condition of the system) in the form

$$\left(\frac{\partial f(x, z)}{\partial x}\right)_{x=0} = 0 \,. \tag{14}$$

The total liquid flow through the system of unit length is given by the general balance equation

$$Q = 2 \int_0^\infty f(x, z) \, \mathrm{d}x + W(z) \,. \tag{15}$$

Solution for the Cases of Dry Wall and Totally Reflecting Wall

We shall use the Fourier method. If we assume that the particular solution has the form

$$f_n(x, z) = X(x) \, Z(z) \,, \tag{16}$$

we obtain the result

$$f(x, z) = \sum_{n=1}^{\infty} A_n \cos(q_n x/a) \exp(-q_n^2 \, \mathrm{To}) \,. \tag{17}$$

Here To denotes a dimensionless criterion[11] corresponding to the Fourier number from the theory of heat conduction,

$$\mathrm{To} = Dz/a^2 \,, \tag{18}$$

and the constants

$$q_n = (2n - 1) \, \pi/2 \,, \quad n = 1, 2, \ldots \tag{19a}$$

denote the zero points of the cosinus function in accord with the boundary condition (*3*). Analogously if follows from (*4*) that for the case of a totally reflecting wall the constants

$$q_n = n\pi\,, \quad n = 0, 1, 2, \ldots \tag{19b}$$

are the zero points of the sinus function, and the distribution function has the form

$$f(x, z) = A_0 + \sum_{n=1}^{\infty} A_n \cos\,(n\pi x/a) \exp\,(-n^2\pi^2\,\text{To})\,. \tag{20}$$

The constants A_n can be determined from the corresponding boundary condition, which is expanded in series according to Eq. (*17*) or (*20*) for $z \to 0$, *i.e.*, $\text{To} \to 0$. The result is

$$A_n = 2\int_0^1 f(x, 0) \cos\,(q_n x/a)\, \mathrm{d}(x/a)\,, \quad n > 0\,, \tag{21a}$$

$$A_0 = \int_0^1 f(x, 0)\, \mathrm{d}(x/a) = Q/2a\,, \tag{21b}$$

where the constants q_n are given by either of Eqs (*19a,b*).

From the above general solution we can easily derive equations for the particular modes of the boundary wetting of the bed and for the given wall behaviour.

Dry Wall; $q_n = (2n - 1)\,\pi/2$

For a generally wetted bed we obtain by combining Eqs (*17*) and (*21a*) and expressing $f(x, 0)$ from the boundary condition (*8*)

$$f(x, z) = 2\sum_{n=1}^{\infty} \cos\,(q_n x/a) \exp\,(-q_n^2\,\text{To}) \int_{x_1/a}^{x_2/a} g(x') \cos\,(q_n x'/a)\, \mathrm{d}(x'/a)\,, \tag{22}$$

since beyond the limits of this integral the function $f(x, 0)$ is equal to zero. If we set in Eq. (*22*) $g(x)$ equal to a constant f_0, we obtain after integration and rearrangement

$$\frac{f(x, z)}{Q/2a} = 2\sum_{n=1}^{\infty} \frac{\sin\,(q_n x_2/a) - \sin\,(q_n x_1/a)}{q_n(x_2/a - x_1/a)} \cos\,(q_n x/a) \exp\,(-q_n^2\,\text{To})\,, \tag{23}$$

for the case of two symmetrical band sources defined by the boundary condition (*9*). Here we used the relation

$$Q = 2(x_2 - x_1)\, f_0\,. \tag{24}$$

The term $Q/2a$ in Eq. (*23*) is equal to the average wetting density; the left-hand side of (*23*) is called the relative wetting density.

If we perform the limit $x_2 \to x_1$ in Eq. (*22*) we obtain an equation for the case of two symmetrical linear sources. The relative wetting density is given as

$$\frac{f(x, z)}{Q/2a} = 2 \sum_{n=1}^{\infty} \cos(q_n x_1/a) \cos(q_n x/a) \exp(-q_n^2 \text{ To}) . \qquad (25)$$

By setting $x_2 = 0$, Eq. (*23*) takes the form corresponding to a band source (characterized by the boundary condition (*11*)):

$$\frac{f(x, z)}{Q/2a} = 2 \sum_{n=1}^{\infty} \frac{\sin(q_n x_1/a)}{q_n x_1/a} \cos(q_n x/a) \exp(-q_n^2 \text{ To}) . \qquad (26)$$

An equation for the case of a linear source, characterized by the boundary condition (*12*), can be derived from Eq. (*26*) by performing the limit $x_1 \to 0$, or from (*25*) by setting $x_1 = 0$. Thus, we obtain for the relative wetting density

$$\frac{f(x, z)}{Q/2a} = 2 \sum_{n=1}^{\infty} \cos(q_n x/a) \exp(-q_n^2 \text{ To}) . \qquad (27)$$

The relative wetting densities for the envisaged source types and given values of the criterion To are shown graphically in Figs 2*a* – *d*.

Totally Reflecting Wall; $q_n = n\pi$

In the case of a totally reflecting wall, we start from Eqs (*19*) and (*20*), where we introduce the constants A_n from (*21a,b*). Since the derivation is analogous to the case of a dry wall, we present only the results.

Generally wetted bed, boundary conditions (*8*):

$$f(x, z) = Q/2a + 2 \sum_{n=1}^{\infty} \cos(n\pi x/a) \exp(-n^2\pi^2 \text{ To}) \,.$$
$$. \int_{x_1/a}^{x_2/a} g(x') \cos(n\pi x'/a) \, d(x'/a) . \qquad (28)$$

Two symmetrical band sources, boundary conditions (*9*):

$$\frac{f(x, z)}{Q/2a} = 1 + 2 \sum_{n=1}^{\infty} \frac{\sin(n\pi x_2/a) - \sin(n\pi x_1/a)}{n\pi(x_2/a - x_1/a)} \,.$$
$$. \cos(n\pi x/a) \exp(-n^2\pi^2 \text{ To}) . \qquad (29)$$

Two symmetrical linear sources, boundary conditions (*10*):

$$\frac{f(x, z)}{Q/2a} = 1 + 2 \sum_{n=1}^{\infty} \cos(n\pi x_1/a) \cos(n\pi x/a) \exp(-n^2\pi^2 \text{ To}) . \qquad (30)$$

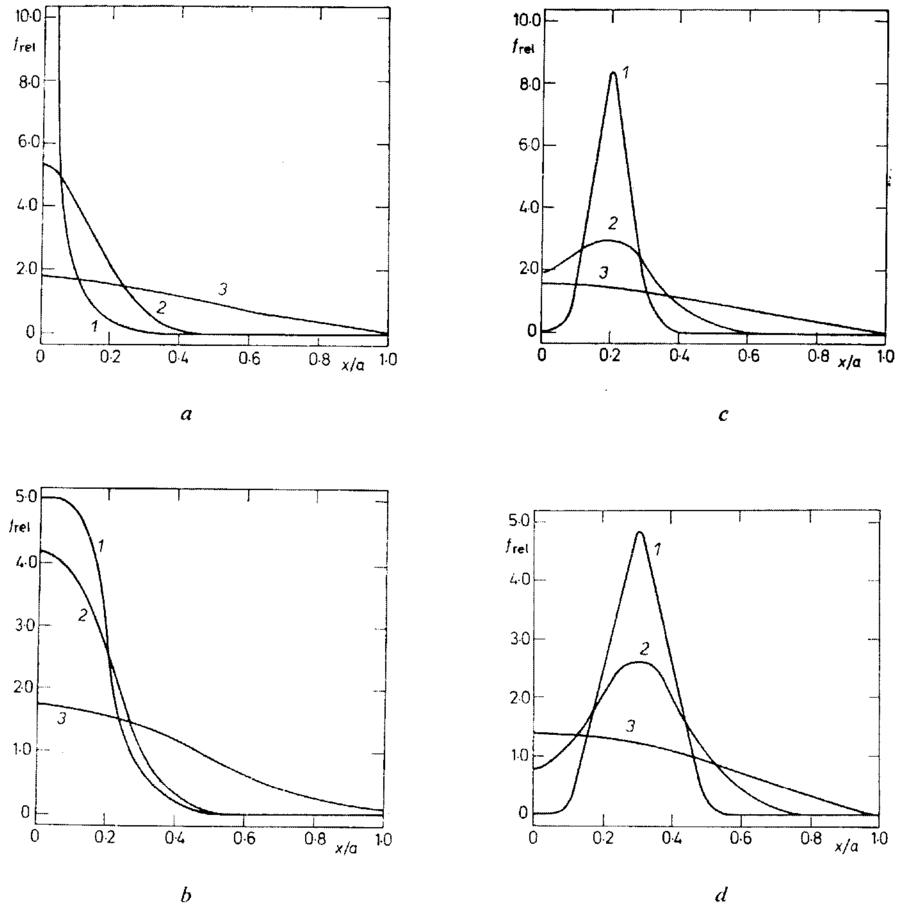

FIG. 2

Dependence of $f_{rel} = f(x, z)/(Q/2a)$ on x/a for the Slit System with a Perfectly Dry Wall

a: Central Linear Source
$f_{rel} = \infty$ for $x/a = 0$; *1* To = 0·001; *2* To = 0·01; *3* To = 0·1

b: Central Band Source
$f_{rel} = 5$ for $|x/a| < 0·2$; *1* To = 0·001; *2* To = 0·01; *3* To = 0·1.

c: Two Symmetrical Linear Sources
$f_{rel} = \infty$ for $|x/a| = 0·2$; *1* To = 0·001; *2* To = 0·01; *3* To = 0·1.

d: Two Symmetrical Band Sources
$f_{rel} = 5$ for $0·2 < |x/a| < 0·4$; *1* To = = 0·001; *2* To = 0·01; *3* To = 0·1.

Band source, boundary conditions (*11*):

$$\frac{f(x, z)}{Q/2a} = 1 + 2 \sum_{n=1}^{\infty} \frac{\sin(n\pi x_1/a)}{n\pi x_1/a} \cos(n\pi x/a) \exp(-n^2\pi^2 \text{To}) . \qquad (31)$$

Linear source, boundary conditions (*12*):

$$\frac{f(x, z)}{Q/2a} = 1 + 2 \sum_{n=1}^{\infty} \cos(n\pi x/a) \exp(-n^2\pi^2 \text{To}) . \qquad (32)$$

The relative wetting densities for these source types and given To values differ from the preceding case only for large To values as can be seen by comparing Figs 3 and 2*a*.

Real Wall

We shall solve the partial differential equation (*2*) for a real wall boundary condition by the method of Laplace transformation. Eq. (*2*) after transformation and using the boundary condition (*13a*) gives

$$D \frac{\partial^2 F(x, p)}{\partial x^2} = pF(x, p) - f_0 , \qquad (33)$$

where $F(x, p)$ denotes the Laplace transform of $f(x, z)$. This equation has the general solution

$$F(x, p) = f_0/p + A_1 \cosh[(p/D)^{1/2} x] + A_2 \sinh[(p/D)^{1/2} x] . \qquad (34)$$

With respect to the symmetry condition (*14*) we have

$$A_2 = 0 , \quad F(x, p) = f_0/p + A_1 \cosh[(p/D)^{1/2} x] . \qquad (35), (36)$$

By combining the equation for the wall flow (*7*) with the boundary condition of the real wall (*5*) and transforming we obtain

$$\left(\frac{D}{\beta} + \frac{2D\gamma}{p}\right)\left(\frac{\partial F(x, p)}{\partial x}\right)_{x=a} + F(a, p) - \frac{\gamma W_0}{p} = 0 , \qquad (37)$$

whence it follows with the aid of Eq. (*36*)

$$A_1 = -\frac{1}{p} \frac{f_0 - \gamma W_0}{(D/\beta + 2D\gamma/p)(p/D)^{1/2} \sinh[(p/D)^{1/2} a] + \cosh[(p/D)^{1/2} a]} . \qquad (38)$$

Now we can introduce this result into Eq. (*36*), define an auxiliary parameter

$$q = (a/\mathrm{i})\,(p/D)^{1/2} \tag{39}$$

and use the known relations

$$\sinh(\mathrm{i}x) = \mathrm{i}\sin x\,, \quad \cosh(\mathrm{i}x) = \cos x \tag{40a,b}$$

to obtain

$$F(x, p) = \frac{f_0}{p} - \frac{(f_0 - CW_0/2a)\cos(qx/a)}{p[(C/q - q/B)\sin q + \cos q]}\,, \tag{41}$$

where the dimensionless criteria B and C are defined as

$$C = 2a\gamma\,, \quad B = \beta a/D\,. \tag{42), (43}$$

The equation for the wetting density is obtained by retransformation of Eq. (*41*). It is obvious that the original of the term f_0/p is the boundary wetting density f_0. The original of the second term on the right can be found with the aid of the Heaviside theoreme: If the Laplace transform of the function $h(z)$ has the form $H_1(p)/pH_2(p)$, then

$$h(z) = \frac{H_1(0)}{H_2(0)} + \sum_{n=1}^{\infty} \frac{H_1(a_n)\exp(a_n z)}{a_n[\mathrm{d}H_2(p)/\mathrm{d}p]_{p=a_n}}\,, \tag{44}$$

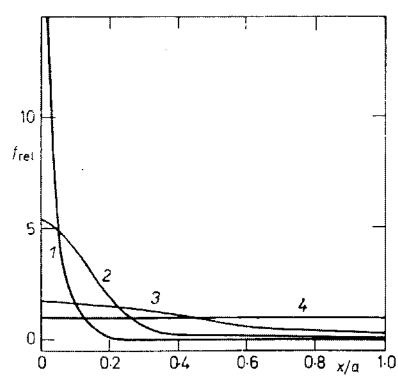

FIG. 3

Dependence of $f_{rel} = f(x, z)/(Q/2a)$ on x/a for the Slit System with a Totally Reflecting Wall and a Central Linear Source

$f_{rel} = \infty$ for $x/a = 0$; *1* To = 0·001; *2* To = 0·01; *3* To = 0·1; *4* To = 0·5.

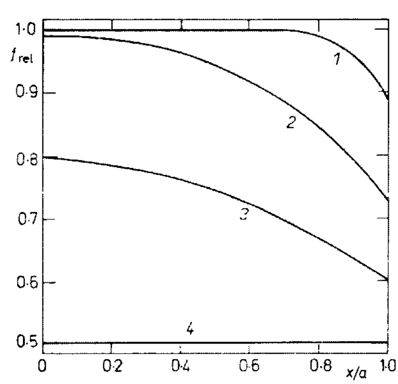

Fig. 4

Dependence of $f_{rel} = f(x, z)/(Q/2a)$ on x/a for the Slit System with a Real Wall and an Equally Wetted Bed

$C = 1$: $B = 1$; *1* To = 0·01; *2* To = 0·1; *3* To = 0·5; *4* To = 7·0.

where a_n denotes simple poles of the ratio $H_1(p)/H_2(p)$. Hence, by retransformation of Eq. (*41*) we obtain

$$f(x, z) = f_0 - \frac{f_0 - CW_0/2a}{C + 1} - (f_0 - CW_0/2a) \, .$$

$$. \sum_{n=1}^{\infty} \frac{\cos(q_n x/a) \exp(-q_n^2 \, \text{To})}{\sin q_n(C/2q_n - 3q_n/2B - q/2) + \cos q_n(1 + C/2 - q_n^2/2B)}, \qquad (45)$$

where q_n is the value of q for $p = a_n$ and is defined by the equation

$$(C/q_n - q_n/B) \sin q_n + \cos q_n = 0 \, . \qquad (46)$$

The roots q_n of Eq. (*46*) for chosen values of the criteria B and C are given in Table I. For $n > 10$ we have approximately $q_{n+1} = q_n + \pi$. If $\sin q_n$ is expressed from Eq. (*46*) and introduced into (*45*) we obtain after rearrangement the resulting equation

$$f(x, z) = \frac{C}{C + 1}(f_0 + W_0/2a) + 2(f_0 - CW_0/2a) \, .$$

$$. \sum_{n=1}^{\infty} \frac{(q_n^2/B - C) \cos(q_n x/a) \exp(-q_n^2 \, \text{To})}{[(q_n^2/B - C)^2 + q_n^2/B + q_n^2 + C] \cos q_n} \, . \qquad (47)$$

For the special case of an equally wetted bed ($W_0 = 0$) we have

$$\frac{f(x, z)}{Q/2a} = f(x, z)/f_0 = C/(C + 1) +$$

$$+ 2 \sum_{n=1}^{\infty} \frac{(q_n^2/B - C) \cos(q_n x/a) \exp(-q_n^2 \, \text{To})}{[(q_n^2/B - C)^2 + q_n^2/B + q_n^2 + C] \cos q_n}, \qquad (48)$$

and for the case of a wall source ($f_0 = 0$)

$$\frac{f(x, z)}{Q/2a} = \frac{f(x, z)}{W_0/2a} = C/(C + 1) -$$

$$- 2C \sum_{n=1}^{\infty} \frac{(q_n^2/B - C) \cos(q_n x/a) \exp(-q_n^2 \, \text{To})}{[(q_n^2/B - C)^2 + q_n^2/B + q_n^2 + C] \cos q_n} \, . \qquad (49)$$

The relative wetting densities for the case of an equal wetting or for the case of a wall source are shown graphically in Figs 4 and 5. The meaning of the individual criteria is obvious from the discussion in ref.[12].

By performing the limit $z \to \infty$ in Eqs (47)–(49) we obtain

$$\frac{f(x, \infty)}{Q/2a} = C/(C + 1). \tag{50}$$

Hence, with respect to the balance of the liquid flow through the system, Eq. (15), we have necessarily

$$W(\infty)/Q = 1/(C + 1). \tag{51}$$

These limiting formulas give a clear meaning to the dimensionless criterion C defined by Eq. (42). With increasing value of C the wall flow diminishes; it is obvious that for $C \to \infty$ the wall becomes totally reflecting.

The criterion B defined by Eq. (43) characterizes the flow of the liquid to the wall; together with the criterion To, which characterizes the horizontal spreading of the liquid through the bed, it determines the distance from the origin where the stationary state is reached.

Based on Eqs (42) and (43), the boundary condition (5) can be written in the form of the criterion equation

$$-\frac{2a}{Q}\left(\frac{\partial f(x, z)}{\partial(x/a)}\right)_{x/a=1} = B\left(\frac{f(a, z)}{Q/2a} - C\,\frac{W(z)}{Q}\right). \tag{52}$$

Measurement of Constants of Distribution Equations

In the preceding communication[1] of this series, the correlation of the criterion To and of the horizontal spreading coefficient D for a cylindrical system was described. Since the value of D is independent of the system geometry, its value measured on the cylindrical system applies also for the slit one. If we compare the meaning of variables in the definition of To for a slit system (22) with the variables occurring in the definition of To for a cylindrical system[11], it follows that the To value for a height coordinate z in a cylindrical system of a diameter $2a$ is equal to that for the same height coordinate z in a slit system of a width $2a$.

The coefficient D or criterion To for a slit system can be measured by the method described in the preceding communication[1] for a cylindrical system.

Previous authors[9,10] who studied the cylindrical system used for the correlation relations for a semiinfinite system of for one with a dry wall. A semiinfinite system is represented by a bed of a finite height h (vertical dimension) and infinite horizontal dimensions. It is characterized by the boundary condition

$$\lim_{x \to \infty} f(x, z) = 0. \tag{53}$$

Tour and Lerman[4] derived from experimental data a relation for the case of a central linear source defined by the boundary condition (*12*). By a suitable choice of a constant, this relation can be rewritten in the form

$$f(x, z) = [Q/2(\pi Dz)^{1/2}] \exp(-x^2/4Dz)\,, \tag{54}$$

which is the so-called fundamental solution of Eq. (*2*), or after rearrangement

$$\frac{f(x, z)}{Q/2a} = (\pi \mathrm{To})^{-1/2} \exp\left[-\left(\frac{x}{a}\right)^2/4\,\mathrm{To}\right]. \tag{55}$$

The symbol a in this equation has from the point of view of the envisaged semiinfinite system no physical meaning. However, it is evident from the definition of the criterion To (Eq. (*22*)) that it represents (as in the preceding cases) the half-width of the bed of the real slit system in which the measurement is carried out. The application of the relationships for the hypothetical semi-infinite system to a real one is possible with the assumption that the width of the real system is sufficiently large to eliminate the influence of the wall on the liquid distribution.

The relative wetting densities for the case of a semiinfinite bed with a central linear source are shown schematically in Fig. 6.

Equations for more complicated modes of the boundary wetting can be derived in a simple way: We shall, *e.g.*, consider a general source defined by the boundary condition (*8*) as a set of linear sources acting in a point x' on the width element dx'. We then obtain the wetting density

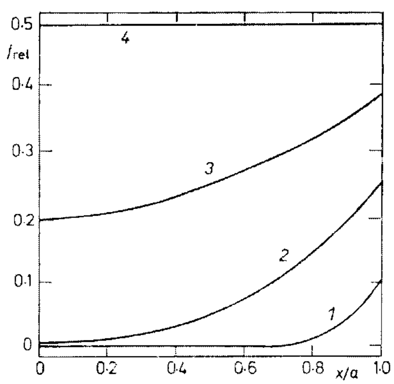

FIG. 5

Dependence of $f_{rel} = f(x, z)/(Q/2a)$ on x/a for the Slit System with a Real Wall and Liquid Led to the Wall

$C = 1$; $B = 1$; *1* To = 0·01; *2* To = 0·1; *3* To = 0·5; *4* To = 7·0.

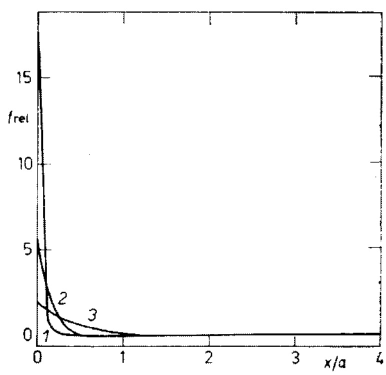

FIG. 6

Dependence of $f_{rel} = f(x, z)/(Q/2a)$ on x/a for a Semiinfinite Bed with a Linear Source

$f_{rel} = \infty$ for $x/a = 0$; *1* To = 0·001; *2* To = 0·01; *3* To = 0·1.

$f(x, z)$ in a given point as the sum of the wetting densities due to the individual linear sources of the set:

$$f(x, z) = \int_{x-x_2}^{x-x_1} [g(x')/2(\pi Dz)^{1/2}] \exp(-x'^2/4Dz)\, dx' + \int_{x+x_1}^{x+x_2} [g(x')/2(\pi Dz)^{1/2}] \exp(-x'^2/4Dz)\, dx' . \quad (56)$$

Numerous solutions for the case of a semiinfinite bed are given for analogous processes (diffusion, heat conduction) in the literature[13,14].

Staněk and Kolář[15] recommend to use for the determination of the horizontal spreading coefficient D equations for a dry wall system. According to them, it is not justifiable to apply equations for a semiinfinite system since in this model of the distribution always a certain fraction of the liquid is for $z > 0$ "beyond the system". (This fraction can be for a unit length of the slit system expressed as

$$\Delta Q(z) = \int_{-\infty}^{-a} f(x, z)\, dx + \int_{a}^{\infty} f(x, z)\, dx , \quad (57)$$

where $f(x, z)$ is the distribution function for a semiinfinite bed.) However, this point of view is disputable with respect to the limited accuracy of the determination of the mentioned constants and to the possibility of neglecting the "flow beyond the system" for relatively small z values (as can be seen from the comparison of the relative wetting densities for a slit system with a dry wall and a semiinfinite system, both wetted by a linear source).

The measurement of the criterion C is based on Eq. *(50)* or *(51)*, which gives the ratio of the liquid flowing through the bed or along the wall for $z \to \infty$ or To $\to \infty$ (Figs 4 and 5).

The criterion B can be determined as in the preceding cases by the methods used for cylindrical systems. In the method of Staněk and Kolář[16], use can be made of an apparatus for the determination of the coefficient D; the flow of the liquid through segments under the bed wetted by an even source or by a wall source is compared with the flows calculated from Eqs *(48)* and *(49)* into which we insert the measured values of To and C and chosen values values of B.

In the method of Dutkai and Ruckenstein[10], use is made of the boundary condition *(5)*. As already mentioned, this equation multiplied by 2 dz represents the balance of the liquid transfer to the wall; hence

$$2\, dz \beta [f(a, z) - \gamma\, W(z)] = dW(z) . \quad (58)$$

If the bed is wetted by a wall source, then

$$\lim_{z \to 0} f(a, z) \doteq 0 . \quad (59)$$

By inserting this relation into *(58)* and rearranging we obtain with the use of *(18)*, *(42)* and *(43)*

$$\ln [W(0)/W(z)] = \ln [Q/W(z)] = BC\, \text{To} . \quad (60)$$

To determine the criterion B (or coefficient β), we therefore measure the wall flow for a sufficiently small bed height and using the criteria C and To (or coefficient γ and coordinate z) we calculate the desired quantity from Eq. (*60*).

LIST OF SYMBOLS

a	half width of the slit system (parallel to the x axis) (L)
A_n	constant
B	criterion characterizing the liquid transfer to the wall
B_n	constant
C	criterion characterizing the stationary state
D	coefficient of horizontal spreading of the liquid in the bed (L)
$f(x, t)$ or $f(x, y, z)$	wetting density (distribution function) (L/T)
f_0	initial wetting density
$f_n(x, z)$	particular solutions for $f(x, z)$ (L/T)
$F(x, p)$	Laplace transform of $f(x, z)$
f_{rel}	relative wetting density
$g(x)$	function of initial wetting density fulfilling the Dirichlet conditions (L/T)
h	bed height (L)
i	imaginary unit
L	length of the slit system (parallel to the y axis) (L)
n	whole number
p	complex variable in the Laplace transformation
q, q_n	constant
Q	total liquid flow through the system of unit length (including wall flow) (L^2/T)
To	criterion characterizing spreading of liquid in the bed
$W(z)$	wall flow in the system of unit length (parallel to the y axis) (L^2/T)
W_0	initial wall flow in the system of unit length (L^2/T)
x, x'	rectangular coordinates (L)
x_1, x_2	source widths in the slit system (L)
$X(x)$	function of coordinate x
y, z	rectangular coordinates (L)
$Z(z)$	function of coordinate z
β	coefficient of liquid transfer to the wall
γ	distribution coefficient (L^{-1})
$\Delta Q(z)$	flow of liquid beyond the system (Eq. (*57*)) (L^2/T)

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